Reg. No. : | | | | | | | | | | |

Question Paper Code : 80873

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2024.

Fourth/Fifth/Sixth Semester

Civil Engineering

MA 8491 — NUMERICAL METHODS

(Common to : Aeronautical Engineering/Aerospace Engineering/Agriculture Engineering/Electrical and Electronics Engineering/Electronics and Instrumentation Engineering/Instrumentation and Control Engineering/Manufacturing Engineering/Mechanical Engineering (Sandwich)/Mechanical and Automation Engineering/Biotechnology and Biochemical Engineering/Chemical Engineering/Chemical and Electrochemical Engineering/Plastic Technology/Polymer Technology/Textile Technology)

(Regulations 2017)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. What is the criterion for the convergence in Newton-Raphson method?

2. Solve by Gauss-Elimation method $2x + y = 3;\ 7x - 3y = 4$.

3. Prove that the relation $E = e^{hD}$.

4. Evaluate : $\Delta^2_{bc}\left(\frac{1}{a}\right)$

5. What is the error in the trapezoidal rule?

6. Write the formula for Romberg's integration.

7. Using Euler's method, find $y(0.01)$ given $y' = -y,\ y(0) = 1$.

8. State the fourth order Runge-Kutta method formula to solve $y' = f(x, y),\ \ y(x_0) = y_0$.

9. Classify the partial differential equation $u_{xx} + 2u_{xy} + 4u_{yy} = 0$.

10. Write Bender-Schmidt's explicit formula for solving heat equation.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Use fixed point iteration method to solve the equation $f(x) = \cos x - 3x + 1$. (8)

(ii) Solve the following system of equations by the Gauss-Seidal method. (8)

$28x + 4y - z = 32;$

$2x + 17y + 4z = 35;$

$x + 3y + 10z = 24$

Or

(b) (i) Apply Gauss-Jordan method to find the solution of the following system. (8)

$x + y + 5z = 7;$

$2x + 10y + z = 13;$

$10x + y + z = 12$

(ii) Find the numerically largest eigen value of $A = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$ and the corresponding eigen vector of the matrix using Power method. Let the initial vector be $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$. (8)

12. (a) (i) Using Newton's divided difference formula, evaluate $f(8)$ and $f(15)$ given. (8)

x	4	5	7	10	11	13
$f(x)$	48	100	294	900	1210	2028

(ii) The following data are taken from the steam table : (8)

Temperature °C	140	150	160	170	180
Pressure kgf/cm^2	3.685	4.854	6.302	8.076	10.225

Find the pressure at temperature t = 142°C and t = 175°C

Or

(b) (i) Using Lagrange's interpolation formula, find $y(10)$ from the following table. (8)

x	5	6	9	11
y	12	13	14	16

(ii) The following values of x and y are given (8)

x	1	2	3	4
y	1	2	5	11

Find the cubic splines and hence evaluate $y(1.5)$ and $y'(3)$

13. (a) (i) Obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 50$, given that (8)

x	50	51	52	53	54	55	56
$y = \sqrt[3]{x}$	3.6840	3.7084	3.7325	3.7563	3.7798	3.8030	3.8259

(ii) Evaluate $\int_1^{1.2}\int_1^{1.4} \frac{1}{x+y}\,dx\,dy$ by trapezoidal rule with $h = k = 0.1$. (8)

Or

(b) (i) Evaluate the integral $\int_0^1 \frac{x^2}{1+x^3}\,dx$ using Simpson 1/3 rule. Compare the error with the exact value. Take $h = 0.25$. (8)

(ii) Using three point Gaussian quadrature formula, evaluate $\int_0^1 \frac{dx}{1+x}$. (8)

14. (a) Determine the value of $y(0.4)$. Using Milne's predictor-Corrector method given $y' = xy + y^2$, $y(0) = 1$; use Taylor series method to get the values of $y(0.1)$, $y(0.2)$ and $y(0.3)$. (16)

Or

(b) If $y' = 2e^x y$, $y(0) = 2$, then find $y(0.4)$ using Adam's predictor-corrector formula, by calculating $y(0.1)$, $y(0.2)$ and $y(0.3)$ using Euler's modified formula. (16)

15. (a) Derive explicit scheme to solve the wave equation and using it to solve $4u_{xx} = u_{tt}$ subject to $u(0, t) = 0$, $u(4, t) = 0$, $u_t(x, 0) = 0$ and $u(x, 0) = x(4 - x)$ taking $h = 1$ (for 4 time steps). (16)

Or

(b) Find by the Liebmann's method the values at the interior lattice points of a square region of the harmonic function u whose boundary values are as shown in the following figure. (16)